SECTION II.—GENERAL METEOROLOGY.

PRESENT STATUS OF OUR KNOWLEDGE OF THE CAUSES OF THE DIURNAL CHANGES IN TEMPERATURE, PRESSURE, AND WIND.

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[At the time of this paper the author, born March 15, 1848, at Neumarkt, Tyrol, was extraordinary professor, but in October, 1893, he was appointed regular professor of cosmical physics at the University of Innsbruck. He remained there, giving brilliant service, until October 1, 1897, when he was appointed professor of terrestrial physics at the University of Vienna and director of the Imperial Central Institute for Meteorology and Terrestrial Magnetism. This institute he greatly enlarged both in size and scope of service, and his exertions there contributed to the causes of his relatively early death at Arco in southern Tyrol December 20, 1908.—C. A. jr.]

The regular daily oscillations that we are accustomed to speak of as periodical are at present known so accurately in reference to the facts themselves that with great confidence we can say that further investigation will not reveal to us anything of importance that is new. Especially has the periodic diurnal variation of temperature and pressure for all regions of the globe in all latitudes and in the most diverse altitudes above sea level, been presented in such detail that even in the minutiæ of the diurnal rate there can scarcely be anything more discovered. This is certainly less true of the winds, but even as to this element important recent discoveries leave nothing more to be expected. It is not now my problem to collect the facts relative to the rate, the diurnal periodic oscillation of temperature of pressure and of the wind; but rather it is my duty to present the present condition of the theoretical explanations of these phenomena. Therefore I assume that in general we already know both the general course and the details of the diurnal variation of the elements under consideration.

I. EXPLANATION OF THE DAILY TEMPERATURE MARCH.

Under the word temperature in general we in meteorology understand the temperature of the air, and under this last expression, if not otherwise expressly stated, we mean the temperature of a stratum of air at about 1 meter above the earth's surface.

We must also state by way of prelude what we understand to be an explanation of a phenomenon. We can say a phenomenon is explained only when we are in a position to give its physical cause; it is perfectly explained only when we can bring this cause into such a mathematical expression that with the help of the latter we are in a position to compute the phenomenon itself.

We recognize with perfect clearness the two very different causes that give the curve of diurnal march of temperature its peculiar form. During each diurnal rotation of the earth the sun is above the horizon so long as it is describing its daylight path, and therefore the heat that is received at any instant is proportional to the sine of the altitude of the sun. Therefore during the daytime the curve of temperature is represented by a sine curve or wave. After sunset the influence of the sun disappears and from the end of the evening twilight to the beginning of the morning twilight it is equal to zero. During the nocturnal path of the sun [terrestrial] radiation only is effective, and this part of the curve of daily temperature march will be, in accordance with the law of terrestrial radiation, an exponential or logarithmic line. The appearance of the curve allows us to easily recognize these two branches; from sunrise to sunset the first branch presents clearly the wavelike elevation of a sine curve; from sunset to sunrise the second branch of the curve steadily descends in the well-marked form of an exponential curve attaining the lowest point at about the time of sunrise. There are, therefore, two causes that determine the diurnal march of temperature, viz, the radiation outward from the earth and the radiation earthward from the sun. So long as the radiation from the sun is effective during the day, it alone determines the form of the curve of temperature, inasmuch as it far overpowers the radiation from the earth which is always present, whereas the latter is the only cause determining the form of the curve during the nighttime.

We therefore know the physical causes of the periodic diurnal changes of temperature and are thus justified in saying that we are able to explain this phenomena.

But there is now a further question whether this knowledge also puts us in a position to give a mathematical expression which enables us to compute adequately the phenomenon and one that can in every case be introduced as the analytical equivalent of the diurnal curve.

In this respect we must at once confess that we are not in a position to formulate a mathematical expression that shall represent the whole curve, both the day branch and the night branch, by means of one formula. But a different answer will be given if we treat the two branches separately, since we are able to give a mathematical expression for the night branch that satisfies the demands of a physical formula, that is to say one in which well-defined physical quantities are utilized.

Lambert (1) was the first who subjected the two branches of the temperature curve to separate treatments. For the night branch he proposed an expression that represented the exponential and is of the same form as we employ to-day. A physical deduction of this expression was first attempted by Weilenmann (2); but he made untenable assumptions in that he referred the whole nocturnal cooling of the air to the cooling of the earth's surface by radiation toward the atmosphere and further assumed that the cooling of the lowest stratum of air was only due to the giving of its heat down to the cooled earth. Maurer (3) then showed that the thermal conductivity of the air did not come into consideration and that the air itself cools directly by reason of its own radiation. On this basis he deduced the formula for the nocturnal temperature march and comes to the same expression that Weilenmann attained notwithstanding his false assumptions, viz,

$$T = T_0 + cb^z$$

[1] The publication of this memoir has been delayed for the reasons given in the MONTHLY WEATHER REVIEW, February, 1914, 42:93.

In this equation z represents the time and b is equivalent to $e^{\frac{\sigma}{\rho c}}$. Thus Maurer gave for b an expression that was physically intelligible in which σ represents the coefficient of radiation of the air, ρ the density of the air, and c its specific heat.

Weilenmann had found that b is a constant for all places and all times. But Maurer objected to this. He made σ refer to the unit of volume and therefore according to his formula b must vary with the density. On the other hand Trabert (4) has shown that the coefficient of radiation is independent of the density and must therefore refer to the unit of mass, viz, that the expression $\frac{\sigma}{\rho}$ used by Maurer represents the constant coefficient of radiation of the air. Hence, b must be a constant for all places and all times unless perhaps the coefficient of radiation of the air depends upon the temperature. But Trabert has shown that the latter case is highly improbable. On the average

$$\log b = -0.066.$$

If now we represent the coefficient of radiation of the air for a unit mass, or one gramme, by σ then from $b = e^{-\frac{\sigma}{c}}$ we obtain for σ the following value:

$$\sigma = -0.547 \log b$$
$$= 0.036 \text{ calories per hour.}$$

On the other hand with regard to T_0, which should represent the temperature of that ideal envelope which we conceive to replace the radiation from all surrounding bodies, Trabert finds from the average of 42 stations distributed over the whole earth that it depends on the average temperature, as follows:

$$t_0 = -3.4° + 1.003 t_m$$

where t_0 and t_m express the values of T_0 and T_m in centigrade degrees.

We have thus obtained for the nocturnal branch of the diurnal temperature curve, a mathematical expression derived from physical causes and in which we understand the physical quantities that occur therein. The formula $T = T_0 + Cb^z$, in which z represents the time expressed in hours after sunset, justifies us in the belief that we can completely explain the nocturnal branch of the temperature curve.

Unfortunately we do not succeed so well in our attempt to explain the daylight branch of the curve of diurnal periodic temperature variation. In order to find a mathematical expression that shall be physically intelligible we must start with the solar radiation as the cause of this branch of the curve. The intensity of the solar radiation after its passage through the atmosphere and for the solar altitude h is expressed by

$$I = Ap^{\epsilon} \sin h$$

and this is the quantity of heat that falls upon the horizontal ground. Now all thermal influences during the daytime in so far as they all depend upon the solar radiation are proportional to this quantity. Among these may be enumerated:

(1) The warming of the air by its absorption of direct solar rays.

(2) The warming by absorption of the rays reflected from the earth.

(3) The warming by absorption of solar rays scattered in and through the atmosphere.

(4) The convection of heat from the ground warmed by the solar rays.

Further we have to recall the negative influences—

(5) The cooling by radiation outward which again is proportional to the temperature of the air.

(6) The cooling by the evaporation of water, which latter again is proportional to the warming by the sun.

The introduction of all these factors into the analytical expression for the daytime temperature march is impracticable for many reasons. Even the coefficient of transmission, p, is variable in a manner too little understood by us to enable us to introduce it correctly; and the other enumerated factors are understood far too little to enable us to express them in mathematical symbols. In consideration of these difficulties to the presentation of a physically intelligible mathematical expression for the daylight branch of the diurnal temperature march, Lambert (5) said long since: "We shall accomplish much if, without too notable error, it can be assumed that the heat which the earth actually receives from the sun is at least proportional to that which the earth could receive if there were no obstacles." To be sure, we know to-day that the latter is the case, but neither this nor the great progress of our knowledge makes the representation of the desired formula any easier.

We can therefore pass over the attempts that have been made in this direction by Lambert himself (6), by Lamont (7), and by Weilenmann (8). However, with reference to the attempt made by Weilenmann it may be said that it appears interesting because his final formula endeavors to present in one expression the whole curve, with its day branch and its night branch. All these formulas, however, give no deep insight into the processes by which Nature determines the periodic temperature change, such as it is desired to obtain from a mathematical expression deduced from the physical causes of the phenomenon. Indeed we fear that these formulas do not contribute to the explanation and the intelligibility of the temperature curve any more than would an ordinary series such as, in the absence of any physical laws, we ordinarily use for presenting any given phenomena.

In fact, Angot (9), leaving aside every theoretical formula, has applied Bessel's Series for the presentation of the diurnal rate of temperature during the daylight hours. Since the form of the daylight branch of the temperature curve suggests a sine wave, and since the formula for the intensity of the rays $I = Ap^{\epsilon}\sin h$ makes it clear that we have to deal with a phenomenon that is approximately proportional to the sine of the altitude of the sun. Therefore among all the series that we could use for the empirical presentation of this daylight branch, evidently the sine series is the most appropriate. If the time of the maximum air temperature coincided with the maximum altitude of the sun, then certainly an empirical formula in terms of the sine of the simple hour-angle of the sun would abundantly suffice to represent the phenomenon. But since the dependence of the curve on the altitude of the sun is not so simple as all this, therefore we must apply many terms of the sine series, such as twice, three times, etc., the hour angle. Angot found, however, that in order to present the daytime formula only, it was sufficient to utilize only terms with twice the hour angle, and with this we obtain a very approximate presentation of the daytime branch of the temperature curve.

We must thus recognize that in regard to the complete explanation of the daytime branch of the temperature curve we are not so fortunate as we have been with the nocturnal branch.

In many cases, especially when it is desired to introduce the temperature rates into a computation, it is necessary to have a formula that brings the curve of temperature for the whole day of 24 hours into one expression. It is evident that we have much less hope of presenting a theoretical expression based on the recognized physical causes for the whole day, than we have for the daylight branch alone. Weilenmann's above-mentioned attempt to do this must certainly be regarded as unsuccessful and a second attempt has never been made. Therefore, in order to meet the above-mentioned need, we must again take refuge in a series, and a series employing the fewest terms possible for an accurate presentation of the phenomenon. Thus we are again thrown back upon the sine series. The complete sine wave consists of a crest and a trough, hence there is a maximum and a minimum. Now in the regular simple sine wave, the maximum and the minimum are equally distant from the zero value that lies between them, but by superpositions of waves having ½-wave-length, ⅓-wave-length, etc., of the original simple wave-length, and further by having different initial times one may so shift the maximum and minimum as to reproduce those actually presented by the diurnal curve of temperature. If, therefore, one employs a sufficient number of the terms of the Bessel Series to represent the curve of the daily temperature march it will be possible to represent the phenomenon exactly. Wild (10) has shown that usually one must go even further than four times the angle to secure sufficient accuracy. In many cases, and especially where the expression for the daily march of temperature is to be introduced in computations, it suffices to take such a number of terms of the Bessel series as will reproduce the general form of the temperature curve. Usually one stops with the term depending upon the 2-fold angle, but I can not refrain from saying that to me this always seems to be too small a number of terms for the proper presentation of the complete diurnal curve. If we do not introduce the 3-fold angle, the form of the curve given by the formula departs too much from the actual curve. In most cases, however, this departure need not be of much importance.

But no matter how many terms of the Bessel formula we introduce, whether we stop at the 2-, 3-, or 4-fold angle, one should never forget that *in nature the daily curve of temperature is not attained by a superposition of these different waves*, the latter are only a mathematical substitute in order to attain by computation to the same curve that Nature has produced in an entirely different manner. We know with certainty how this peculiar curve is produced in nature: During the daytime the sun, or Nature, produces a simple sine curve, but only the upper half of the wave; during the nighttime, however, radiation produces the continuation of the curve and draws a logarithmic line. Only the maximum, therefore, belongs to a wave of the sine-curve type, the minimum has nothing whatever to do with such a wave. If, however, we determine to construct the whole curve out of sine-waves the latter is only a mathematical device and we should never forget that it is so.

I have thought that this exposition should be given because this seems to me to be the correct physical conception of a question that is, moreover, worthy of a fundamental discussion. I will now summarize the present condition of our knowledge of the explanation of the diurnal periodic variation in temperature in the following paragraphs:

(1) We know that the curve of this variation consists of two entirely distinct parts, the daylight and the nocturnal branch.

(2) We know the physical causes that give to each of these branches their peculiar form. The daylight branch is due to the radiation from the sun, the variable warming depending on the sine of the sun's altitude. The nocturnal branch owes its form to the cooling by radiation from the earth. Therefore the former is the crest of a sine-curve and the latter is the steady fall of a logarithmic line.

(3) We are enabled to represent the nocturnal branch by a mathematical formula deduced from theoretical considerations on the basis of well-known physical causes.

(4) We are not also in a position to deduce such a formula for the daylight branch and must therefore seek to represent it for computational purposes by means of the sine curve, i. e., by Bessel's series, of which Angot finds that, even for the daylight branch, it is sufficient to include only the 2-fold angle.

(5) Still less do we possess a physically established theoretical formula to represent the whole diurnal curve. We can, however, in the absence of such a formula (which would give one maximum and one minimum only) again take refuge in Bessel's series, but now in order to attain an accurate representation of the true curve, we must utilize a development to at least the 4-fold angle.

II. EXPLANATION OF THE DIURNAL RATE OF PRESSURE.

As the temperature curve of the diurnal change is characterized by one maximum and one minimum, so the most prominent character of the diurnal curve of pressure is the twofold maximum and the twofold minimum. In the temperate zone there occurs also a very small third maximum, but as this is not general it can not be reckoned as a general property of the pressure curve. The knowledge of the course of these phenomena is to-day as perfect as can be desired (11). But to the question what are the physical causes of the diurnal change of pressure, we are not yet in position to give any satisfactory answer. In comparison with the answer that we can give relative to the diurnal curve of temperature, we can say very little as to the daily pressure march. In this case we know not the physical causes that determine this march, and still less do we know the laws in accordance with which we should devise a theoretical mathematical formula based on physical causes, in order to represent the phenomenon itself. Hence this part of my problem must be very quickly answered by admitting that we do not know the physical causes of the diurnal change of pressure, and therefore also have no mathematical expression that satisfies the requirements of a physically intelligible formula.

But in this way I should but illy present the present condition of the question. Since the discovery of the barometer there has been scarcely a single prominent meteorologist but has interested himself in the solution of this problem. Therefore in order to present the present condition of our knowledge on the subject, I must at least collect the hypotheses, whether good or bad, that do not embody anything preposterous and that have found either general or limited acceptance.

Of all those explanations that have been thus far suggested, Dove's theory (12) on the pressure of the dry air is the only one that has been given up.

With one single exception, all assume as the basis of the diurnal curve of air pressure, the diurnal changes in temperature. Ramond (13) has applied this latter phenomenon in the simplest manner to the explanation of the diurnal variation of pressure. He assumes that the heating of the atmosphere by the sun, makes itself felt for the meridians that lie between 9 a. m. and 3 p. m. Over this whole region the atmosphere expands and overflows toward the west and the east, whereby the minimum of the afternoon and the two maxima, morning and evening, are explained. In order to explain the nocturnal minimum, Ramond assumes that there is an increase of temperature about midnight.

Espy (14), in order to explain the principal maximum of 10 a. m., introduced a new cause, viz, the tension imparted to the lower stratum of warmed air, since although warmed it can not immediately expand. With reference to the nocturnal minimum, Espy is still more unfortunate than Ramond. Kreil (15) retained Espy's idea and added a further active cause in the shape of ascending and descending currents of air to explain our phenomenon. The first maximum he explained according to Espy, the afternoon minimum according to Ramond and with the help of the ascending currents, while the evening maximum was cited as a consequence of the descending current. His explanation of the nocturnal minimum was interesting. The lower strata, compressed in consequence of the descending current, must expand on the cessation of this current, and thus again there arises an ascending but very weak current that leads to the nocturnal minimum.

The Espy theory of tension was opposed by Lamont (16), but Blanford (17) and Angot (18) later defended it.

Rykatchev (19) gave a very different explanation. He made the diurnal variation of the pressure depend upon the regular diurnal changes in both the upper and the lower air currents. He put the increase and decrease of the atmospheric pressure proportional to the difference of the velocity of the transportation of the air per unit of longitude, and then by the summation of the effect of the upper and the lower currents of air he comes to his explanation of the diurnal variation of pressure. Where the velocity of the wind increases as it advances, it evidently leaves thinner air behind it; where with increase of progress it diminishes its velocity it produces a damming up or accumulation and increases the density of the air. The summation of these effects of the upper and the lower currents of air is complicated and rather forced. Rykatchev describes the process, as he understands it, quite fully and explains in detail how according to his views the two maxima and two minima must be developed.

Liais (20) also brings in a new idea to explain our phenomenon. He points out the fact that the air ascending in the locality where it is most strongly heated, must remain behind toward the west in consequence of the inertia of its ascent, and thus contributes to the formation of the forenoon maximum. On the other hand, in the late afternoon and early hours of evening, the descending air must flow forward toward the east, and thus exert a pressure that contributes to the formation of the evening maximum. In other respects Liais must be considered as the predecessor of Rykatchev, since with the help of the just mentioned item he attempted to explain the diurnal march of pressure by reason of the horizontal air currents due to the warming.

If we now add the remark that since the time of Dove the diurnal increase and decrease of vapor pressure also has usually been adduced in explanation of the increase and diminution of atmospheric pressure, we may say that we have spoken of all the physical causes that have been used to explain the diurnal change of pressure. Only one quite isolated attempted explanation still remains to be mentioned.

Peltier (21) believed that the electric condition influenced the pressure of the air. The earth is negatively electrified; the vapors that rise from the earth are also negatively charged. At the time of the most rapid evaporation the electric repulsion is also the greatest; the barometer falls. But when the vapors attain higher altitudes they become positive toward the earth, and there results an attraction; the barometer rises. If in consequence of cooling, a condensation of vapor occurs in the lower stratum then the barometer falls; if the cooling proceed further and eventually the upper strata sink lower, then the barometer must again rise.

According to all this, we can certainly not say that too little has been done to discover the physical causes of the diurnal change of pressure; in fact, there is not a single cause at work in the atmosphere or acting upon it that has not been seized upon. We have taken these causes individually, combined and accumulative, and must confess that in fact there has been no want of effort to attain the object desired; and still all these efforts at explanation remain unsatisfactory. There is not a single one of them of which the majority of meteorologists would say that it suffices; not one of these solutions is of such a kind that it gives us confidence that the correct solution has been found or indeed that the true cause of the phenomenon even as to its principal cause has been found. We have only found that the temperature is one of the principal factors. We find only that most of the adduced causes may contribute many details to the actual form of the diurnal curve of pressure; the fundamental cause, properly so called, we must state has not yet been recognized. If the recognition of the fundamental causes of the diurnal change in pressure seems so unsatisfactory, on the other hand, in reference to a theoretical formula deduced mathematically from the physical causes to represent this diurnal curve of pressure, no attempt has been made. Those who have presented the above attempted explanations are satisfied to describe verbally the influence of their assumed causes without computing, even approximately, the magnitude of the effects to be expected.

Under such circumstances one need not be surprised that it was preferred to give first a more accurate and analytical description of the diurnal curve of pressure for the whole globe, in order thereby when possible to secure a better basis for a mathematico-physical theory of the phenomena as Hann (22) has expressly stated. The process to be chosen is given by the curve of diurnal pressure. In the most regular form of this curve as it occurs in the Tropics and also in the annual averages for the temperate zones, having two maxima 12 hours apart and two minima also 12 hours apart, it points so clearly to the double sine curve that we are involuntarily led to its mathematical presentation by means of the Bessellian sine series.

It is to be expected, considering the nature of the curve, that the term depending on the 2-fold angle must be the most important and that the whole phenomenon will find its basis therein, whereas disturbing influences will be shown by the other terms.

Lamont (**23**) has already adopted this method of mathematical description. Of course, some had already used this method of presentation of the diurnal pressure curve by means of the Bessel series. Lamont, however, limits himself to the terms of the single and 2-fold angle and is only able to show that the amplitudes of the 2-fold waves are of remarkable magnitude and depend only on the geographic latitude, as also that the epochs of these double waves is the same over the whole earth and agrees with the times of maximum and minimum and coincides with the times of maxima and minima as read directly from the diurnal curve of pressure. On the other hand he shows that the term with the simple angle is the more variable. The term with the 2-fold angle shows itself independent of temperature, cloudiness, rainfall, and other local influences all of which occur plainly in the first term. Hence he calls the double wave the "atmospheric flood and ebb," but, of course, excludes from this conception all influences of gravitation. Still he could find no other explanation of the great regularity and the independence of terrestrial processes of this double wave except that it is ascribed to a cosmic force which he preliminarily would call an "electric" force. In his studies of this subject Lamont had only a very limited number of stations at his disposal.

Hann (**24**) and Angot (**25**) have helped us in this respect, for in their investigations they have taken into consideration many more than a hundred stations distributed over the whole earth. They completely confirm Lamont's results and even give them greater precision. Hann represents the amplitude of the double wave, in its dependence on the latitude, by the following equation:

$$a_2 = -0.222 + 1.184 \cos^2\phi$$

with regard to which he remarks that slight irregularities are still to be seen therein. Lamont also had pointed out such small irregularities in the double wave. Angot has sought to eliminate these irregularities under the conviction that the term depending on the 2-fold angle does not depend on any terrestrial accident. He holds that the influence of temperature as expressed in the term depending on the simple angle is, strictly speaking, expressible only by a series of several terms (as is certainly not to be doubted); therefore the term depending on the 2-fold angle is a complex one that consists of a double wave of pressure independent of temperature, with superposed terms depending on the 2-fold angle of the temperature. He separates these two numerically and retains the purely constant term of the double wave of pressure. Of course this can not be done without being arbitrary to a certain extent. Following La Place's presentation of an ebb and flow, he expresses the amplitude of this principal wave of pressure (onde semidiurne principale) by the formula:

$$a_2' = \frac{A \cos^2\delta}{r^2}$$

where δ is the declination of the sun, r is the radius of the earth's orbit. He then determines A, the amplitude, from the observations and finds the expression:

$$A = 0.926 \frac{b}{760} \cos^4\phi$$

where ϕ is the latitude and b is the average pressure of the air. He thus arrives at the following expression for the "semidiurnal principal wave" of atmospheric pressure:

$$y = 0.926 \frac{b}{760} \cdot \frac{\cos^2\delta}{r^2} \cdot \cos^4\phi (2x + 64°).$$

This latter would therefore be the expression for the constant double wave of pressure resulting from a "cosmic cause."

We must recognize that by these investigations of Hann and Angot our knowledge of the phenomenon of the variation of pressure has been advanced to an extraordinary extent. We now know with certainty that the double wave of pressure is a constant phenomenon, so constant that we think only a cosmic cause can offer a satisfactory explanation thereof. The causes that can enter as disturbances occur in the remaining terms of the sine series. Nevertheless we must recognize the fact that a physical explanation of the phenomenon has not yet been found. In order that it may be found it will be necessary on the one hand that we state the "cosmic cause" in order that when possible we may deduce from it a mathematical expression that shall represent the phenomena. Equally do we now know that temperature, cloudiness, rainfall, and local influences do have disturbing influences on the course of the principal wave, but we have as yet no expression deduced from their physical properties that formulates the effect of these disturbing influences. Therefore we can not to-day speak of a physical explanation of the daily range of temperature.

Quite recently [1890] Margules (**26**) made the only attempt known to me relative to such an explanation. He develops mathematically the idea of Sir William Thomson (**27**), now known as Lord Kelvin. Sir William [2] says:

> The explanation probably is to be found by considering the oscillations of the atmosphere, as a whole, in the light of the very formulas which Laplace gave in his Mécanique Celeste for the ocean, and which he showed to be also applicable to the atmosphere. When thermal influence is substituted for gravitational, in the tide-generating force reckoned for, and when the modes of oscillation corresponding respectively to the diurnal and semidiurnal terms of the thermal influence are investigated, it will probably be found that the period of free oscillation of the former agrees much less nearly with 24 hours than does that of the latter with 12 hours; and that therefore, with comparatively small magnitudes of the tide-generating force, the resulting tide is greater in the semidiurnal term than in the diurnal.

Margules carried out the computation and found Thomson's prediction justified. This temperature-wave progressing from meridian to meridian successively in a rotating spherical shell which is considered similar to the atmosphere, has a whole-day period of the form

$$\tau = C \sin\omega \sin(nt + \lambda),$$

where τ is the temperature, ω the polar distance, λ the geographic longitude, n the rate of rotation of the earth, t the time. It produces for the temperature $T_0 = 273°$ a pressure-wave having the form

$$\varepsilon = C \sin(nt + \lambda)\,[1.146 \sin\omega - 0.423 \sin^3\omega - 0.370 \sin^5\omega - \ldots]$$

[2] As the original English seems to have passed through a series of translations, the Editor takes the liberty of reprinting the original words of Kelvin on page 400 of the Proceedings of the Royal Society of Edinburgh, Vol. XI.

On the other hand, temperature-waves having a ½-day period and the shape corresponding to the equation

$$\tau = C \sin^2\omega \sin(2nt+2\lambda)$$

produce pressure-waves having the form corresponding to the equation

$$\varepsilon = -C \sin(2nt+2\lambda)[37.99 \sin^4\omega + 23.06 \sin^6\omega + 5.75 \sin^8\omega + \ldots]$$

The factor $\sin^4\omega$ jumps from $-\infty$ over to $+\infty$ when the temperature is near $T_0=268°$. On the other hand, if the temperature of the atmospheric spherical shell is nearly 268°A., then slight semidiurnal temperature-waves are able to produce large pressure-waves in air that is devoid of friction. We should thus have a physical explanation of the diurnal change of atmospheric pressure that we can restate as follows: The physical cause of the diurnal change of atmospheric pressure is to be found in the diurnal change in temperature; the principal pressure-wave is the semidiurnal, and is produced by the semidiurnal wave of temperature. Although this last is small, yet it produces a considerable semidiurnal wave of pressure because for an average temperature of about 268°A. the atmosphere has very nearly the period of the forced vibration that closely approximates its free vibration and this is not true of the 24-hour oscillation of temperature. On the basis of this knowledge it would then become possible to construct the correct mathematical expression for the diurnal curve of pressure.

But many doubts attach to this explanation. Margules himself says:

Its application to the diurnal barometric variation is clear in one respect only; the semidiurnal wave of pressure can also be considered as the consequence of a semidiurnal temperature-wave of small amplitude. This theory explains the relative magnitude, but not the uniformity of the semidiurnal wave of pressure above the land and the ocean. * * * The neglect of the friction and of the vertical motion of the air in our last computation, *the assumption of a uniform average temperature in the whole mass of air* and the assumption of the equality of the oscillation throughout any given latitude, allows us to make only a vague application of this prinicple.

And thus we come back to the original point of view that Margules expresses in his introduction: "If we consider this semidiurnal wave of pressure as a consequence of the changes of temperature, still the connection is very obscure."

I can not resist expressing the most obscure feature of this matter, viz, that the very existence of a semidiurnal wave of temperature is not proven or even made probable. I need not repeat what I have said at the conclusion of the chapter on the variation of temperature and the interpretation of Bessel's sine series. I need only emphasize the fact that we have no right to attribute to the one-half daily wave of temperature a reality that does not belong to it, since it is only a mathematical auxiliary, in order to arrive by pure mathematics at the construction of a curve that nature arrives at in another way well known to us by means of the temperature. And yet only an actual double daily wave of temperature having two actual maxima and minima can be the physical cause of the actually existing double daily wave of pressure.

Must we, therefore, give up the temperature wave as a cause of the double diurnal pressure wave and again return to an at least equally obscure "cosmical cause" as its origin?

On this point I allow myself to sketch some thoughts which appear to me to be worth a closer examination, and that I hope to be able to give thorough and accurate attention in some other place. If we consider the earth with its atmosphere as a unit, then there is always one whole hemisphere warmed by solar radiation, and the quantity of heat that the earth receives from the sun is always the same—excepting for changes of the sun itself and the periodic annual changes of the distance of the sun from the earth. The illumined hemisphere considered as a whole will, therefore, always receive a nearly constant amount of heat. This is the basis of the remarkable constancy of the amplitude of the semidiurnal wave of pressure which has repeatedly led to the assumption of a "cosmical cause." We thus have this constant cosmic cause before us. How does the semidiurnal wave of pressure arise, since the sun comes to each meridian only once every day? I think of this as analogous to the tides of the ocean. The moon also passes through the meridian but once a day; but we know that the tide arises simultaneously in the meridian and 180° therefrom and we know why, and that thus the semidiurnal tide is brought about. As to the double wave of atmospheric pressure, something similar occurs: On the hemisphere that is heated directly the surfaces of equal pressure in the atmosphere are elevated—most strongly in the region where the warming is the greatest—therefore air flows toward either side toward the hemisphere that does not receive radiation from the sun. Hereby a mass of air is drawn from the illumined hemisphere and carried over to that which is not illumined. Now, the consequence of this shifting of masses on the earth would directly produce a shift in the position of the center of gravity of the whole system, earth–air. According to a well-known law in mechanics the center of gravity of a free floating body is invariable, a change of position of certain free masses on the body produces a compensating change of position of other free masses. Under our conditions then, according to this theorem, masses must flow away from the region 180° distant and the accumulation of masses must also be so arranged that they will attain their maximum at 180° from each other and at 90° from the places whence the masses flow. Thus the change of the center of gravity is prevented. Thus we should have two maxima and two minima of pressure that must continually circulate around the globe with "cosmic" regularity.

Most of the causes that have been adduced by different meteorologists in explanation of the diurnal change of air pressure, very probably do bring about various modifications. Thus I hold it to be fairly certain that the morning maximum becomes the principal maximum by reason of the tension [see above p. 658] conceived of by Espy and by several others down to Angot. Again the horizontal motions of the air make their influence felt and possibly also the dynamic pressure of the sinking atmosphere during the evening maximum comes into consideration. In short, the individual processes above mentioned may all be plausibly considered as modifying causes. But I will not here enter into these details. The idea of a mechanical explanation for the "cosmic" regular semidiurnal wave of pressure briefly sketched above can easily be further extended by anyone. It seems unnecessary to multiply words on the subject. If I am able to compute the relative magnitudes that result from this cause, then I will on some other occasion endeavor to give a detailed theory of the diurnal changes of pressure based upon the above-mentioned foundation.

This report on the present condition of the theory of the daily periodic pressure changes must unfortunately be rather elaborate because no well-established theory exists, but from what has been stated one can really attain a complete picture of the condition in question.

I will summarize my report as follows:

The diurnal temperature variation is the fundamental cause of the diurnal oscillation of the barometer. The "cosmic" regularity of the semidiurnal wave of pressure has its origin in the solar heat which is constant to the whole earth; and has its mechanical cause in the theorem of the constancy of the center of gravity of free, floating bodies. A number of consequences from the diurnal march of temperature, such as the horizontal and vertical air currents, evaporation, the tension of air warmed at the surface of the ground, etc., may modify the original wave of pressure. A physically intelligible theoretical mathematical expression based upon the physical causes of the diurnal curve of pressure is still lacking. A sine series of Bessel that is used instead of this deductive formula reproduces the phenomenon very exactly when we develop it to the 3-fold angle and gives us especially a good insight into the character of the semidiurnal wave of pressure that we have recognized as the principal wave of pressure.

III. EXPLANATION OF THE DIURNAL CHANGE OF THE WIND.

In reference to the wind, we have to consider two elements for the investigation of its diurnal changes—the direction of the wind and the velocity of the wind.

It is necessary to remark at the outset that the condition of our knowledge of the facts relative to this diurnal change is considerably worse than for temperature or pressure of the air, and corresponding thereto we have a smaller number of attempts to explain the phenomena. In order not to be too verbose, we must here also restrict ourselves to the explanation of the phenomenon and assume its details to be well known.

One peculiarity occurs in the case of the winds that we must take cognizance of first. The wind is a motion of the air, and in such motions the resistances to motion play an important part. In our case the friction of the wind against the earth and the obstacles presented by its various topographic forms is the cause of many peculiarities. We here have to do with a stratum of air adhering more or less to the surface of the earth, which evidently must have its own peculiar conditions. At an altitude above the earth that is not especially high—at any rate less than 300 meters, as shown by the observations on the Eiffel tower—the free movement of the air is demonstrably present.

Since we are primarily concerned with a general problem of the atmosphere, we will first consider the unrestricted moving air, that is to say, the diurnal period of the direction and velocity of the wind of the higher free strata of air. We will then consider the diurnal period of the air in the lowest strata near the earth in all its peculiarities and be in a position to understand them as a result of the phenomena in the upper strata and the friction of the lower strata at the earth's surface.

This may seem like a new method of treating the subject, and it may be that I shall depart somewhat thereby from my problem to depict the present state of the question; but we shall find an accurate presentation of the matter, although I must also say, by way of introduction, that we have little knowledge of the subject.

A. DIURNAL CURVE OF THE WIND IN THE UPPER AIR STRATA.

1. *Direction of the wind.*—I have, as I think, demonstrated (28) that undoubtedly "the wind changes with the sun." The explanation of this phenomena I find in the fact that where the sun stands in the meridian the column of air is raised to the greatest extent, and therefore in the strata above the isobaric surfaces must also be raised so that through the whole column there is a barometric gradient from this region of greatest warming toward the regions of lesser and least warming. In the morning hours the sun stands in the east, which is therefore the region of greatest warming. Thus there arises a gradient from the east and in each case, if other causes have not caused a stronger gradient in some other direction, easterly winds will arise. In the evening this region of strongest warming lies to the west of us, and under the same conditions we have westerly winds. For the temperate latitudes and about midday, however, a south-to-north gradient must develop because the greatest warming then takes place in the south, and thus under these conditions south winds must prevail. Therefore, day after day the wind goes round with the sun, as the observations also demonstrate.

2. *Wind velocity.*—The diurnal march of wind velocity is ordinarily computed without regard to the direction of the wind. In such cases it is certain that the minimum must fall somewhere near the midday hour and the maximum velocity near midnight.

But the question remains whether this holds good for every wind no matter what its direction. I have investigated this question (29) without coming to any certain conclusion. It is quite possible that the diurnal march, without regard to the direction of the wind, may be only the resultant of the diurnal march of the totality of the individual wind directions. In fact, I found for the summit stations Sonnblick, Säntis, Obir, Pikes Peak, Pic du Midi, and Puy de Dôme, a rather variable time of occurrence of the maximum in particular, and the minimum also, for the different wind directions. According to the above given explanation of the daily march of wind direction, a different diurnal march is to be expected for each different direction of the wind, such that the east wind should have its maximum in the morning hours, the south wind at the time of maximum temperature, and the west wind in the evening. For it is clear that any gradient that produces an east wind will be increased most decidedly in the morning, whereas during these hours a gradient that produces a west wind will be most enfeebled. Similarly, a south wind considered as a consequence of the general distribution of pressure, will be strongest at midday; a similar west wind will be strongest in the evening, each being a consequence of the increase of the corresponding gradients for these hours. But this explanation does not completely correspond to any one of all the mountain stations investigated by me. The Sonnblick station is the only one that shows the anticipated time of beginning of the east, south, and west winds, but even then only for the simple wave of the diurnal march represented by the Besselian series. On the other hand, the amplitude of the 2-fold wave shows such a large value—it is indeed much larger for the east wind than the value for the simple wave—that this result also appears to be less satisfactory. At first glance the diurnal march for the different directions of the wind shows that except for the south wind on the Sonnblick, particularly the minimum always occurs near midday. Therefore, after the investigation of the diurnal march of wind velocity for the individual wind directions in general, there remains a curve such as shows the minimum about midday, whereas the maximum occurs sometimes during evening, sometimes at night, sometimes during the morning hours.

Hence I conclude that two causes determine the diurnal march of the wind velocity: (1) For the east, south, and west winds, the above-given increase of the gradient during the morning, midday, and evening, respectively. To this cause also I ascribe the different hours of the occurrence of the maxima. (2) A disturbing cause occurring uniformly about midday for all winds. This latter disturbing cause I believe to be the friction which occurs in the upper strata by reason of the warmed ascending air. As is well known, it has been shown by several, and especially by von Helmholtz (30), that the internal friction of the air is an entirely negligible quantity when we consider smooth surfaces of separation (glatte Trennungsfläche), but that this becomes appreciable when two layers flowing at different velocities send streamlets (Strömchen) into each other. The latter process occurs particularly during the noonday hours in the upper strata of air, and in consequence thereof the velocity of the upper air decreases at that time.

I do not deny that this explanation may in general agree with that given by Koeppen (31) and Sprung (32), although these two investigators also attributed the delay to the *slower-moving* lower air masses that are ascending. But after all in this case also "friction" will also come out as a hindrance.

Another question is, how high in the free atmosphere this influence will make itself felt. On the Eiffel tower it is decidedly present, and this is easily explained by the intrusion of air ascending from the nearby surface of the earth. On the mountains the lower air ascends to the top as a valley wind, but whether at altitudes comparable with the Säntis and Sonnblick interchange of air in the free atmosphere with the lower strata still exists, seems to me very doubtful. As I believe, the diurnal march of wind velocity for the individual wind directions at those altitudes must be simply and only determined by the march of the sun itself, as was the case with the wind direction. Therefore I hold fast to the conclusion that this is also the principal cause of the diurnal march of the wind on mountain summits, and that the midday friction explained above is only a modifying cause.

B. DIURNAL CURVE OF WIND IN THE LOWER STRATA OF AIR.

In this case I do not separate the treatment of the wind direction from the wind velocity because they both seem to me to allow of a single explanation.

As far as concerns the wind direction, it is in general true at all stations for which hourly observations are at hand, and where the land- and sea-breeze of the coast or the mountain and valley winds in the interior do not exercise a local control, the following theorem holds good: "The wind changes with the sun the same way as in the upper strata." This must seem quite remarkable. Not less peculiar does it appear that the diurnal march of wind velocity for all directions rises throughout the day to a maximum, but during the night time sinks to a minimum (33).

Now it seems to me that the explanation of these two phenomena is to be sought in the fact that in both cases the lowest strata of air regulate their behavior by the influence of the upper strata. If we first disregard all temperature influences then, during the nighttime, the lower strata of air are held by friction with the surface of the earth and thus possess a much smaller velocity than the air masses flowing smoothly above them. During the day the lower strata are warmed, they both expand and also send their streamlets (Strömchen) into the upper strata; and in consequence of the resulting friction between the upper and lower strata they are dragged along by the upper layer and set into a more rapid motion. Hence in the upper strata there is produced a diminution, but in the lower strata an increase in velocity; and thus is explained the maximum of wind velocity in the lower strata at the time when the upper strata have their minimum. This also explains the nocturnal minimum of the lower strata, because then the friction is absent and they are left to themselves.

Precisely as with the velocity of the wind, so also the wind direction is determined during the day by the upper strata and for the same reason. Thus we may explain the fact that the wind direction in the lower strata also "changes with the sun." Here also we must recognize the fact that Köppen and Sprung (34) have also referred this phenomenon to the influence of the upper strata, but they suppose the more rapidly moving air sinking from the upper stratum to be its cause. There are, however, two points to be considered here: First, the march of wind direction does not agree with that demanded by their theory (35); and second, as to the wind velocity, the following modification has been made: since the observations on the Eiffel tower have been available it is scarcely to be assumed that the "lower" strata—the ones we are here concerned with—extend up as much as 100 meters above the ground. This lowest stratum will, to be sure, not only send air upward during the warming of the morning hours, there must also be air descending from above; but as the warming progresses the sinking air will not reach the lowest stratum since, in consequence of its expansion, the warmer air below while transferring a portion to the upper stratum will require a decreasing quantity to replace the loss because it itself requires more space below as it expands under the general spread of the warming up. Much more might be said with regard to this process, but all leads to the same result, that we can no longer appeal to descending air entering the lowest strata precisely at the time of the lower maximum in that stratum. This is the reason why I adhere to the explanation I gave above for this phenomenon of the diurnal march of the wind in the lower stratum of air.

SUMMARY.

It would seem unnecessary to attempt any more profound words relative to the main point of my problem, viz, to bring out the mechanico-physical connection between the phenomena of the daily periodic march of temperature, pressure, and wind in the atmosphere. Everything that has hitherto been said shows its dependence on the march of temperature which itself again depends on the radiation from the sun. And thus we of course again are driven to the central cause of all physical processes in our planetary system.

NOTES AND REFERENCES.

I. *Temperature.*

(1) **Lambert.** Pyrometrie, pp. 322 and 141.

(2) **Weilenmann** in Schweizerische Meteorologische Beobachtungen. 1872, 9 Jhrg.; p. XLVII.

(3) **Maurer** in Schweizerische Meteorologische Beobachtungen. 1885, 20 Jhrg.

(4) **Trabert.** Der tägliche Gang der Temperatur und des Sonnenscheins auf dem Sonnblickgipfel. Denkschriften der math.-naturw. Classe der K. Akad. d. Wiss., Wien [dated 15. October 1891], 59. Band, p. 177, ffg; p. 205.

Maurer assumed that the coefficient of radiation of the atmosphere, σ, referred to the unit of volume and was therefore independent of the density, ρ. Consequently he expressed the differential equation by

$$\rho c \partial T = \sigma(T - T_0)\partial z,$$

the integral of which is

$$T = T_0 + Cb^z, \text{ where } b = e^{-\frac{\sigma}{\rho c}}$$

From the observations on the Sonnblick, Trabert computed the value of b and found it as large as for the stations on the lowland. If b were independent of ρ, then it must have been much smaller. Other high stations also show that b is independent of ρ. Hence Trabert concluded very properly that the coefficient of radiation referred not to the unit volume, but to the unit of mass, and was therefore independent of ρ. Therefore the differential equation should read

$$e\partial T = (T - T_0)\sigma\partial z,$$

and the integral should be

$$T = T_0 + Cb^z$$

where b has the value $e^{-\frac{\sigma}{c}}$ which is, of course, slightly different from its value in the previous equations.

But it is now remarkable that b varies with the time of year, and therefore we arrive at the hypothesis that it varies with the temperature. This dependence upon temperature must, however, be recognizable by the comparison of tropical stations with those in higher latitudes. Trabert investigated 42 stations by groups, distributed over all latitudes, and did not find any dependence of b on the temperature. He explained the annual change of b by the method of computing b. *See* **Trabert,** "Die Wärmestrahlung der Atmosphäre." Met. Ztschr. 1892, **27**:41.

(5) **Lambert.** Pyrometrie, p. 322.

(6) **Lambert.** Pyrometrie.

The final form which Lambert gives to the formula for the daylight portion of the diurnal curve is as follows:

$$\frac{2y}{\cos p \cos \delta} = 2\sin\varphi + \sin\omega + \cos\omega + \sqrt{2}\sin(45 - \varphi)e^{-(\frac{1}{2}\pi + \varphi + \omega)}$$

where ω is the hour angle counted from noon (local mean time), e is the altitude of the equator, p is the altitude of the pole or the latitude of the place, δ is the declination of the sun, and φ the true anomaly of the sun. According to this formula he computed a table of the values of $\frac{2y}{\cos p \cos \delta}$ for the successive lengths of the day between 6 and 18 hours. In order to obtain the ordinate y, the values of that table must be multiplied by $\frac{1}{2}\cos p \cos \delta$. This value of y is therefore nothing else than the elevation of the ordinate above that abscissa which passes through the minimum value. The accuracy of the value thus obtained leaves much to be desired.

(7) **Lamont.** Darstellung der Temperaturverhältnisse auf der Erdoberfläche, p. 10.

The Lamont formula reads:

$$t = l + pz + q\cos(z + \epsilon);$$

this is a purely empirical formula, since he made no attempt to express physically the important terms in the radiation.

(8) **Weilenmann** as above quoted, puts the rate of warming equal to ap^ϵ and the rate of cooling equal to kp^ϵ, and thus made all quantities proportional to the radiation. Thus the warming of the ground was

$$bp^\epsilon \cos\xi;$$

where ξ is the zenith distance; the cooling of the ground by convection was

$$\lambda p^\epsilon \cos\xi;$$

the cooling of the ground by heat radiated to the air was

$$h(t' - t);$$

and the cooling of the ground by radiation of heat to the upper strata of air was

$$h(t' - u);$$

in all which formulas t = the temperature of the air, t' = the temperature of the ground, u = the temperature of the upper air strata, and z = the time. His differential equations read

$$\frac{\partial t}{\partial z} = (a - k)p^\epsilon + h(t' - t)$$

$$\frac{\partial t'}{\partial z} = (b - \lambda)p^\epsilon \cos\xi - h(t' - t) - h(t' - u).$$

By the combination of these two equations Weilenmann found

$$\frac{\partial^2 t}{\partial z^2} + 3h\frac{\partial t}{\partial z} + h^2 t = h^2 u + p^\epsilon(a\pi\frac{\partial \epsilon}{\partial z} + 2h\alpha + h\beta\cos\xi)$$

$$\alpha = a - k; \text{ and } \beta = b - \lambda.$$

The integral of this equation is not to be taken smoothly. By various manipulations Weilenmann brings it into the following form

$$t = u + [c_1 + c_2(z + b^{Z+z})]b^Z + (\xi_1 + \xi_2\sin Z + \xi_3\cos Z)p^\epsilon;$$

where Z is the semidiurnal arc, and the constants u, c_1, c_2, b, are to be determined by the nocturnal observations, and the constants ξ_1, ξ_2, ξ_3, and p are to be determined by the daylight observations. One can not say that the equations are easily handled and their deduction dose not imply their correctness.

(9) **Angot.** Sur la variation de la temperature à Paris. Annales, Bureau central météorologique de France. Year 1888, I.—Memoires, page B135.

Angot is certainly the only one who has systematically separated the daylight and nocturnal branches in the computation.

(10) **Wild.** Die Temperaturverhältnisse des russischen Reiches. p. 4–6.

II. *Atmospheric pressure.*

(11) We certainly owe this knowledge first of all to the industrious observers who have been and still are active for a long series of years over the whole earth. We are also greatly indebted to many meteorologists who have worked over and published these observations. But our chief gratitude goes out to those who have collected the scattered observations and have discussed them from a single point of view; among these we must mention above all others Rykatchev, Hann, and Angot.

Rykatchev. La marche diurne du baromètre en Russie. Repertorium für Meteorologie, t. **6**, No. 10.

Hann. Untersuchungen über die tägliche Oscillation des Barometers. Denkschriften der K. Ak. d. Wiss., Wien, 1889, math.-naturw. Cl., Bd. **55**, p. 49.

Weitere Untersuchungen über die tägliche Oscillation des Barometers. 1892, Bd. **59**, p. 297.

Angot. Étude sur la marche diurne du baromètre. Annales, Bur. central mét. de France, Year 1887, I—Mémoires, p. B237.

For the high stations we must also add:

Pernter. Ueber den täglichen und jährlichen Gang des Luftdruckes auf Berggipfeln und in Gebirgsthälern. Sitzungsb., K. Ak. d. Wiss., math.-naturw. Classe, II. Abth., Wien, 1881, **84.**

(12) **Dove,** in Poggendorffs Annalen, 1831, **22:** 219; 1842, **58:** 177; and Sitzb., preuss. Akad., Berlin, 1860, p. 644.

(13) **Ramond.** Mémoires sur la formule barométrique de la mécanique céleste et les dispositions de l'atmosphère qui en modifient les propriétés. Clermont-Ferrand, 1811, pp. 89–92. (Troisième mémoire lu à la Classe des sciences physiques et mathématiques de l'Institut les 5, 12 et 26 décembre, 1808. Imprimé dans les mémoires de la Classe, année 1808, 2e semestre, p. 73.)

The paragraph in question, translated from the French, reads as follows:

Considered from this point of view, the phenomenon seems to me to be susceptible of a very satisfactory explanation. While the sun is in our meridian it warms that part of the globe comprised between the places of its apparent rising and setting. Let us suppose that this warming becomes sensible from the 9 a. m. circle to the 3 p. m. circle. The air expands; the surface of this portion of the atmosphere rises above the level of the neighboring layers and discharges some of its excess upon them. The barometer falls, but at the same time it necessarily rises in the regions comprised between the circles of 3 p. m. and 9 p. m., and between the circles of 3 a. m. and 9 a. m., for within these two regions the air is condensed by the cold of the morning and of the evening; the surface of the atmosphere is depressed and this depression is filled little by little by the overflow from the higher layers of the two neighboring regions. Thus the movement is propagated step by step and communicates itself to that part of the atmosphere that is embraced between the two nocturnal circles. The barometer falls, therefore, from 9 p. m. to 3 a. m. because the air has lost its density by the diminution of the cold which took place in the middle of the night and has lost its height by reason of the tribute that the higher layers have paid to the two limiting regions.

(14) **Espy** *in* Report of the British Association, 1840 (Glasgow), Notices and abstracts, page 55.

He says:

"When the sun rises the air begins to expand by heat; this expansion of the air, especially of that near the surface of the earth, lifts the strata of air above, which will produce a reaction causing the barometer to rise; and the greatest rise of the barometer will take place when the increase of the heat in the lower parts of the atmosphere is the most rapid, probably about 9 or 10 a. m."

With reference to the nocturnal maximum, Espy says:

"As the barometer now stands above the mean (at 9 or 10 p. m.), it must necessarily descend to the mean at the moment when it is neither increasing nor decreasing in temperature, which will be a little before sunrise."

(15) **Kreil.** Ueber die täglichen Schwankungen des Luftdruckes. Sitzb. d. K. Ak. d. Wiss., Wien, **43**: 121.

Kreil states that he will try "to explain the diurnal variation of barometer by the changes of temperature without utilizing any other cosmical force."

"Of course we must consider not merely the different temperatures, but also all other direct and indirect consequences of the increasing and diminishing temperatures, such as the greater or less *tension* of the inclosed and compressed air masses; the *elasticity* of air in consequence of which when pressed on one side it does not immediately set up a progressive motion, but is at first compressed on the side of the pressure; its *inertia*, according to which when once air is set in motion it continues in motion even when the cause of the motion no longer exists; the heat which the ground absorbs and radiates outward; and above all the ascending and descending motion of the air masses."

On page 131 he summarizes all this as follows:

At sunrise the lowest stratum of air, by reason of the cooling of the earth's surface, finds itself in a condensed and by the sinking of the upper strata of air, also in a compressed condition. The tension of this lowest air is further increased by the gradual warming of the ground and the air, hence the pressure on the barometer increases still further and in fact until the ascending air current produced by the sunshine attains such strength that the diminution of pressure produced by it overcomes the increase of tension produced by the increasing temperature. This is the instant of the maximum, and from this point onward the atmospheric pressure diminishes and more rapidly in proportion as the ascending current is stronger.

This is Kreil's explanation of the morning maximum and the afternoon minimum. The evening maximum is attributed to the descending current of air. His explanation of the nocturnal minimum is quite peculiar, and is as follows:

But the atmosphere in this condition is again not in equilibrium; for, by reason of the cessation of the motion from above downward, the lowest, compressed air strata acquire an excess of force that must manifest itself by pressing upward against the air that rests upon it. Thus arises again an upward motion that, although very much slighter than that of the previous morning, must bring about a diminution of atmospheric pressure and lead to the minimum that occurs after midnight.

(16) **Lamont** *in* Sitzungsb. der bayerischen Akad. d. Wiss., 1862, 1: 95, ffg.

(17) **Blanford** *in* Proc., Asiatic Society of Bengal, 1876, No. VIII, p. 176; Proc., Royal Society, London, **44**: 411. In this latter memoir he says:

But in 1876 * * * occurred to me that Lamont's assumption, that the atmosphere is free to expand vertically, lifting the superincumbent mass, is subject to an important modification which may greatly alter the conditions of the problem as contemplated by him.

These conditions take no account of the resistance to expansion that must be opposed by the highly attenuated but extremely cold external atmospheric strata of great but unknown thickness, the existence of which is proved by the phenomena of luminous meteors.

(18) **Angot.** Étude sur la marche diurne du baromètre, [see (11)], p. 342:

Starting with sunrise, the lower layers of air warm up rapidly and tend to rise; but this movement will only be produced later when the warming shall have been sufficient to be communicated to the upper regions of the atmosphere; in the first hours after sunrise the lower layers remain in place, warm up and expand, whence an increase in their elastic force and the more rapid rise of the barometer.

(19) **Rykatchev.** La marche diurne du baromètre en Russie [see (11)].

He analyzes the winds into their N–S and E–W components and eventually attains the following results:

During the daytime, viz, from 8 or 9 a. m. up to 5 or 7 p. m., the air moves from west to east; during the nighttime the motion is from east to west; the air moves with greater velocity by day than by night.

To-day we know how erroneous all this is, and we have explained this in chapter III on the diurnal curve of the wind. Rykatchev founds his theory on this assumption. He says:

Thus above each unit of surface of the ground the mass of air will increase up to the mean result by $\frac{24k}{2\pi}\Delta v$; consequently the total pressure of the atmosphere on the unit of surface will augment to an extent proportional to Δv; hence the variations in pressure produced by the lower air currents are proportional to the variations of the velocity component in the east-west currents.

Having come to a similar conclusion with regard to the upper currents, he arrives at the following more complicated theory:

The morning maximum.—From 6 a. m. up to 2 p. m., the lower current contributes to the rarefaction of the air and to the diminution of pressure; during this same time in the upper layer the warm current from the east becomes stronger and stronger; the warmed-up air accumulates more and more above the lower, comparatively cold air; at the commencement the increase of pressure, produced by the current of the upper air, is greater than the diminution caused by the current of the lower air, and the sum total of these pressures increases up to 10 a. m., when the barometer attains its maximum. From this moment the pressure of the upper layer steadily continues to increase, but slowly; the influence of the lower layer becomes predominant and the barometer commences to fall.

The afternoon minimum.—On the approach of the hour of maximum temperature, the warm current from the east in the upper layer begins to diminish to such a degree that a greater quantity of air flows along this layer toward the west than toward the east. The pressure depending on the current of the upper layer diminishes; the current from the west in the lower layer contributes to the fall of the barometer up to 2 p. m.; then it acts in a contrary direction, but after attaining its maximum velocity the current varies but little in velocity and its influence is feeble, the barometer continues to fall until 4 p. m. After this the influx of air by the lower current reacts upon the diminution of the air produced by the upper current.

The afternoon maximum.—The current from the west in the lower air becomes more and more feeble; soon it is replaced by the current from the east and then the velocity increases until nightfall; consequently during all this time the current of the lower stratum increases the pressure. In the upper layer at the moment of maximum temperature the air's inertia keeps it moving from the east, this current diminishes in velocity and its direction changes. This movement of the air from the east, commencing feebly, increases in velocity; thus the pressure of the upper layer diminishes; but until 10 p. m. the influence of the current of the lower layer is predominant and the barometer rises; after this time it is the current of the upper layer which exerts the greater influence on the trend of the barometer; the total pressure of the atmosphere commences to diminish.

The nocturnal minimum.—Passing afternoon maximum, the pressure of the upper layer continues to diminish until 4 a. m.; the current of the lower layer during the night exerts almost no influence on the trend of the barometer; during all the time from midnight to 6 a. m. its velocity is almost constant; the barometer falls until 4 a. m.; afterward in the upper layer the current from the east is delayed by the opposing current which replaces it and becomes stronger and stronger; then the barometer ought to rise.

(20) **Liais.** Théorie mathematique des oscillations du baromètre. Paris, Bachelier. 1851.

Whoever, misled by the title of the pamphlet, expects a mathematical treatment of the causes that are enumerated will be disappointed.

(21) **Peltier.** Recherches sur la cause des variations barométriques Académie Royale de Bruxelles. Extrait du tome XVIII des mémoires cour. et mim. des savants étrangers.

Peltier described the fundamental principle of his explanation on page 66, saying: "One of the more immediate effects of these electric attractions and repulsions is to form an atmosphere either heavier or lighter, and consequently they are the cause of the numerous variations that the atmospheric pressure experiences."

(22) **Hann.** Untersuchungen über die tägliche Oscillation des Barometers. See (11) page 52, where he says:

"(This investigation) will have for its main object nothing more than a strict scientific description of the atmospheric tides, and thus form the foundation for a subsequent mathematico-physical theory of them."

(23) **Lamont,** *in* Sitzungsb. d. bayerischen Akad. d. Wiss., 1862, Bd. 1, page 113, writes:

Whereas in the previous table, representing the first term of Bessel's sine series, there resulted an excessive influence of the locality; we find here, in the term for the 2-fold hour-angle, a remarkable agreement as to epochs, and a regular diminution in the magnitude of the motion from the Equator to the poles, whence there can be no doubt that here we have to do with a general phenomenon conditioned only in a very slight degree upon the locality.

And on page 118:

These tables give, as I believe, the complete demonstration as to the correctness of my proposed explanation for the diurnal motion of the barometer, inasmuch as on the one hand they show that the first term increases and diminishes in exact harmony with the monthly curve of the temperature of the air and therefore appears as the effect of the temperature, while the second term—whether we consider northern or southern, higher or lower stations—always has the same form and both because of its double period in 24 hours as well as on account of its independence of the season of the year, can not be ascribed to any direct or indirect influence of temperature.

And again on page 122:

I consider these facts, together with the facts above mentioned, as a distinct demonstration that the ebb and flow of the atmosphere must be ascribed to some cosmical force.

These last thoughts he develops still further in the Bulletin de Bruxelles, classe des sciences de l'Académie royale 1859, t. **26**. On page 137 he says:

Now what is the force that produces this regular motion of the atmosphere? As it is evident from the first that the effect is due to a direct or an indirect action of the sun, the first force to which we would naturally attribute it is gravitation which produces a similar movement in the layer of water that covers the earth. * * * But as the movement due to lunar attraction is only $\frac{1}{10}$ of a Paris line near the Equator, it is impossible that the much weaker action of the sun could produce a movement of more than $\frac{1}{4}$ a Paris line. * * * One is finally led to the conclusion that the heat of the sun together with all that depends upon it, can not explain the oscillations of the barometer and that it is necessary to attribute these to a force similar to that of gravitation which, like it, produces in a fluid layer covering the surface of the globe the same effect on points diametrically opposite. Among those forces, whose existence has been recognized or assumed, there is only a single one that meets this condition: and this is the force of electricity which is undoubtedly manifested in cometary phenomena. In fact, let us assume that the sun possesses a great quantity of positive electricity, and that this electricity acts upon an isolated fluid sphere. The two electricities will be separated within the sphere by induction and the hemisphere that is turned toward the sun will be attracted while the opposite hemisphere will be repelled, so that the whole fluid sphere will assume an oval form. Thus the action of the sun's electricity will produce in our atmosphere an effect similar to that which gravitation produces in the waters of the ocean, and the same force that produces the diurnal motion of the magnetic needle will serve to explain the diurnal oscillations of the barometer.

(24) See the memoirs quoted under (2).

Hann has devoted a great number of additional studies, either wholly or in part, to the investigation of the periodic diurnal variations of pressure. This is not the place to enumerate all these; but I would refer to the brief memoir:

Hann. Bemerkungen zur täglichen Oscillation des Barometers. Sitzungsb. K. Ak. d. Wiss., math.-naturw. Cl., Wien, 1886, **93 II**: 981.

In this memoir Hann throws out a new thought toward the explanation of the constant semidiurnal wave of atmospheric pressure. He asks whether possibly the absorption of the radiant heat from the sun by the upper strata of the air can be the source of this wave and of its constancy.

It is easy to perceive that by the periodic diurnal influence of the solar rays on the upper layers of the atmosphere occurring similarly day after day, periodic motions of great regularity must arise in the upper strata of the atmosphere, viz, an oscillation of the whole mass of the atmosphere. These motions can explain the typical character of the diurnal oscillations of the barometer, whereas the local differences represent the basis of the element that modifies the result.

We here see that Hann had correctly appreciated the *amplitude* of the double wave of pressure.

In this elegant little memoir it is interesting to perceive the demonstration that the magnitude of the amplitude of this wave has nothing to do with sun-spots, whence Hann correctly draws the following conclusion:

This diurnal oscillation of the pressure can not depend on the electricity of the sun, as was thought by Lamont, for in that case it must certainly have a period in common with the magnetic variations which evidently depend upon the sun-spot period.

(25) **Angot.** Étude sur la marche diurne du baromètre. [See (11).] On page B311 he says:

An examination of the figures in Table 4 shows that the semidiurnal wave is a complex wave resulting from the interference of two distinct waves. One of these, which we shall call the *secondary semidiurnal wave*, presents one maximum and one minimum in the course of the year like the diurnal wave and, like it also, is influenced by local conditions. * * * This secondary wave is then certainly due, like the diurnal wave, to the diurnal variation of the temperature of the lower layers of the atmosphere. The second wave, which we shall call *the principal semidiurnal wave*, presents very different characteristics; its amplitude experiences a double variation in the course of the year; it is a maximum at the two equinoxes, and a minimum at the solstices. * * * One can indeed already foresee that the phase of this second wave for one and the same station is constant throughout the year.

Again on page 338 Angot says:

The diurnal curve of the barometer can be considered as the resultant of the superposition of two waves having very different origins and characters. One of these waves is independent of the special geographic conditions of each station; it depends only on the position of the sun in its orbit and on the latitude.

After Angot has remarked that perhaps it may be possible that this "semidiurnal, principal wave" also has a term containing the 4-fold angle ($\psi+4x$), but in that case its amplitude certainly can not be even 0.02 to 0.03 mm., he then proceeds to say:

The second wave can be represented by a series such as

$$a_1 \cos(x+\psi_1)+a_2 \cos(2x+\psi_2)+a_3 \cos(3x+\psi_3) \ldots$$

This wave is caused, at least in great part, by the diurnal variation of temperature in the lower layers of air, and consequently all its coefficients depend not only on the latitude and the season but equally on the particular situation of each station; the coefficients change their values with every change of condition and every local influence that can modify the diurnal variations of temperature; we are therefore justified in calling this second wave by the name "thermal wave."

(26) **Margules.** Ueber die Schwingungen periodisch erwärmter Luft. Situngsb., K. Akad. d. Wiss., math.-naturw. Kl., Wien, 1890, **99**: 204–227.

Translated in—

Abbe, C. Mechanics of the earth's atmosphere. Washington, 1891. (Smithsonian misc. coll. No. 843.) pp. 296–318.

(27) **Thomson, Sir W.** On the thermodynamic acceleration of the earth's rotation. Proc. Royal soc., Edinburgh, 1882, **11**: 400. *See* Margules (26), page 207.

III. *Wind.*

(28) **Pernter.** Die Windverhältnisse auf dem Sonnblick und einigen anderen Gipfelstationen. Denkschr., Kais. Akad. d. Wiss., Wien, **58**: 209, ffg.

(29) **Pernter.** Idem, pages 206 and 207.

(30) **Helmholtz.** Ueber atmosphärische Bewegungen. Sitzungsb., Kgl. preuss. Akad. d. Wiss., Berlin, 1888. *Reproduced in* Meteorologische Zeitschrift, 1888, **23**: 329.

Translated in—

Abbe. Mechanics of the earth's atmosphere. Washington, 1891. (Smithsonian misc. coll., No. 843.) pp. 78–93.

(31) **Köppen,** in his remarks on Hann's great work:
"Die tägliche Periode der Geschwindigkeit und Richtung des Windes." Sitzungsb., d. Kais. Ak. d. Wiss., 2 Abth. Wien, **89**: 11 ffg.; *also in* Meteorologischen Zeitschrift, **14**: 343; and more extensively in Annalen d. Hydrographie, **11**: 625.

(32) **Sprung.** Lehrbuch der Meteorologie, page 341.

(33) Almost every trace of variation in the diurnal curve is lacking on the ocean.

(34) **Sprung.** Deutsche Meteorologische Zeitschrift, **1**: 15.

(35) The many additional items added by Sprung by counting the rotation of the windvane, are based in general on observations made only three times a day, and this insufficient observational material may certainly explain the result attained by him.

ON THUNDER.[1]

By WILHELM SCHMIDT.

[Dated K.k. Zentralanstalt für Meteorologie u. Geodynamik, Vienna, 1914.]

1. From earliest times a thunderstorm, and particularly the thunder and lightning, has made the greatest impression on man. It is, therefore, all the more strange, that precisely these phenomena have remained so little studied, and that our knowledge of the sound phenomena has not been increased by more experiments that are something more than analogies. And yet it is not at all difficult to secure results in this field. Observations that may be made when it thunders, themselves point the way to such experiments. Beside the extremely violent, usually deep-toned peals—though they sometimes have a clear ringing or a rushing sound—one may also hear the ringing or breaking as of window panes accompanying some heavy thunder crash; the vibrations can even be perceived by the sense of touch, and sometimes by the trembling of the ground. Thus phenomena whose intensity far exceeds that producible by sound, demonstrate that other vibrations than the audible ones are also present. The very depth of the tone leads to the assumption that there are yet deeper toned pressure variations of such few vibrations that they are inaudible and the direct cause of the effects mentioned. We shall, therefore, endeavor to demonstrate these vibrations which are something quite novel in nature, as well as to complete the picture by recording the audible vibrations.

METHODS FOR RECORDING THE VIBRATIONS.

2. *Instrument I.*—Two different instruments, I and II, serve to accomplish these two purposes. The instrument, I, designed to record the longer vibrations could use a mechanical registration since there was considerable energy available and the velocities to be recorded were not too great. In its final form, I consisted of a wooden box of 210 liters capacity, having all its joints carefully sealed, and with a hexagonal aperture of more than 250 sq. cm. in one of its sides. This aperture was almost closed by a very light aluminum plate suspended by means of two long threads, so that the plate could swing freely in and out. Thus every atmospheric compression-wave falling upon the box must also compress and reduce the volume of the air within the chamber and the aluminum plate, acting as a piston, swing inwards. The reverse process was caused by a rarefaction of the air. A simple train of levers was sufficient to transmit these vibrations to a recording pen writing on the moving sheet of a chronograph. Experience with seismographs shows that the best device is the endless strip of smoked paper running over a motor-driven cylinder upon which rests the recording point of the pen arm. The band of record paper is stretched by a free roller suspended in the lower loop and set at a slight angle with the driven cylinder thus causing a lateral shifting of the record strip and a spiral record. With a slight friction it was possible to secure recording speeds of 5 to 8 mm. per second. The record was fixed by means of a shellac solution in the usual way.

3. *Standardization of instrument I.*—It would be a mistake to assume that the displacement of the pen is proportional to the variations in pressure. The inertia

[1] Author's abstract (German) of the two following papers: "Analyse des Donners," Sitzungsb., K. k. Akad. d. Wissens. IIa, Wien, 1912, **121**: 2045.
"Ueber das Wesen des Donners," ibid., IIa, 1914, **123**.
Translated from Meteorologische Zeitschrift, Braunschweig, Okt. 1914, **31**: 487–498.—C. A. jr.